Creep behavior of poly(methyl methacrylate) with growing damage

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Abstract

A simple optical technique is used to quantify creep crazing damage in a uniaxially stressed transparent glassy polymeric sheet of polymethyl methacrylate (PMMA) at room temperature. The areal craze densities are taken as a measure of the crazing damage. It is shown that the crazing damage increases with nominal applied stress and creep time. The crazing initiation time and the crazing damage evolution equation for PMMA is modeled from a mechanical viewpoint and the results are used to predict the nonlinear creep behavior of PMMA with growing damage. The predictions are in good agreement with the test results.

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1. Introduction

Strengthening and toughening of polymers have received increasing research attention since the 1980s as a result of a better understanding of their deformation and failure mechanisms. Crazing is usually a precursor of the brittle fracture of glassy polymers. Craze is a small crack-like defect and has some load-bearing capacity due to the stretched fibrils connecting the opposite faces of the craze. Among the experimental techniques used to study the crazing behavior of polymers, the most often used are the scanning electron microscope or transmission electron microscope, the optical interference technique, the low angle electron diffraction and small angle X-ray scattering. Normally, these experimental techniques are used to investigate the craze microstructures [1]. From a mechanical point of view, crazing is a type of damage behavior experienced in glassy polymers. Despite considerable advances [1–10], sufficient efforts have not been made to characterize the crazing damage variable. Tang et al. [8] adopted the dilatation of the representative volume element to characterize crazing damage, and found that the damage variable varies linearly with the total strain. G’Sell et al. [11] also investigated crazing damage using a variation of the volumetric strain. In this paper, a simple optical technique is used to quantify creep crazing damage, which is assimilated to the areal craze density, on a uniaxially stressed transparent glassy polymeric sheet of PMMA at room temperature. The crazing damage evolution is formulated and used in predicting the creep behavior of PMMA with growing damage.

2. Crazing damage model

Crazing is a time, temperature and stress dependent damage behavior in polymers. In our tests, a specified constant load is applied to a poly(methyl methacrylate) sheet, whose dimensions are shown in Fig. 1, for a specified period of time at room temperature, \((23.5 \pm 0.5\,^\circ\text{C})\). The specimens are then immediately observed under an optical microscope, at regularly spaced positions along the specimen axis (dot-dashed line in Fig. 1). All the captured surface images that reveal crazing damage are saved in computer for further analysis. Because these images are captured at different sites along the axis of the specimen, they correspond to different nominal stresses due to the varying cross-sectional areas of the sample. The areal craze density, which is defined as the ratio of the crazed area to the whole image area, is calculated through image analysis. Fig. 2 shows the test results [12]. It is shown that for a given test time, the crazing damage increases with applied stress in an approximately linear manner. Based on a linear regression of the test data, an extrapolation of the data yields the craze initiation stress as a function of loading time. It is found that increasing constant applied stresses correspond
to shorter crazing incubation times, as shown in Fig. 3. There are, therefore, two different critical stresses at a given temperature. One is the so-called critical crazing initiation stress, \( \sigma_0 \), below which no crazing occurs at any time, and the other is the critical fracture stress, \( \sigma_f \), at which the material fractures instantaneously, leading to a zero crazing initiation time. So we suppose that the crazing initiation time is of the following form [12]:

\[
t_0 = C \left( \frac{\sigma_f - \sigma}{\sigma - \sigma_0} \right)^q, \quad \sigma \in [\sigma_0, \sigma_f].
\]  

For the PMMA material investigated here, \( \sigma_f = 56 \text{ MPa} \) and \( \sigma_0 = 10 \text{ MPa} \). Fitting Eq. (1) to the experimental data of Fig. 3 yields \( C = 1400 \text{ s} \) and \( q = 2.5 \).

From Fig. 2, one can also get the time dependence of crazing damage at a fixed stress level, as shown in Fig. 4. One can see that the areal craze density increases with time for a fixed stress value, at a rate that increases with stress for a fixed value of time. Under creep loading, the following crazing damage evolution law was derived [12]:

\[
D(\sigma, t) = \frac{1}{m+1} \left( \frac{\sigma - \sigma_0}{\sigma - \sigma_0} - 1 \right)^n \left( \frac{t}{t_0} - 1 \right)^{m+1},
\]

\[
( t \geq t_0, \quad \sigma > \sigma_0 ),
\]

(2)

where \( D(\sigma, t) \) is the crazing damage variable, and \( m \) and \( n \) are stress-dependent non-dimensional material parameters which are nonlinearly related to the normalized stress, \( \sigma/\sigma_0 \). Performing a nonlinear fit of Eq. (2) on the test data of Fig. 4, one obtains the following fitting functions for \( m \) and \( n \):

\[
m = 4691.5 \exp \left( -3\frac{\sigma}{\sigma_0} \right) - 0.52778
\]

\[
n = -53.01479 + 50.19235 \left[ 1 - \exp \left( -\frac{\sigma}{\sigma_0} \right) \right]
\]

(3)

(4)

Substituting Eqs. (1), (3) and (4) into Eq. (2), one obtains an explicit evolution law for crazing damage of PMMA under creep loading, as a function of stress and time.

3. Creep prediction with growing crazing damage

In this section, the creep behavior with growing damage of PMMA under high stresses is predicted, using the stress-dependent damage evolution law given above, from the test performed under lower stresses at which case no damage occurs. Regular type I specimens whose dimensions satisfy the requirements of ASTM D 638 were submitted to different constant loads inducing nominal stresses of 25, 30, 35 and 40 MPa in the gauge section. The time-dependent deformations and the corresponding apparent creep compliances are shown in Figs. 5 and 6. It can be seen that the creep compliance curves are deviate from each other after certain times, indicating that the creep behavior is nonlinear for stresses beyond 25 MPa. At the stress level of 25 MPa, no crazes are observed on the specimen surfaces during the 8000-s test duration. This coincides with the prediction of the damage evolution law,
Assuming that the nonlinear mechanical behavior of PMMA is only induced by the crazing damage, then the creep strain observed as damage increases can be expressed as follows according to the strain equivalence concept in continuum damage mechanics [13]:

$$\varepsilon(D, t) = J(D, t) \sigma = J^0(t) \sigma_{\text{eff}} = \frac{J^0(t) \sigma}{(1 - D)},$$

(5)

where $J(D, t)$ is the apparent creep compliance function of the damaged material under stress $\sigma$, $J^0(t)$ denotes the compliance function of the material without damage, $\sigma_{\text{eff}} = \sigma/(1 - D)$ defines the effective stress. In this study, the measured compliance curve at 25 MPa is used to predict the creep behavior coupled with growing damage based on Eq. (5). Using the crazing damage evolution in Eq. (2), the predictions obtained from Eq. (5) are plotted in Fig. 5 (full lines). It is shown that for stresses below 35 MPa, the predictions agree well with experimental data, while the prediction for 40 MPa is slightly below the experimental value. This may be due to some other coupled nonlinear mechanisms occurring at high stresses.

4. Conclusion

In this study the evolution of crazing damage was investigated in a transparent PMMA sheet under uniaxial creep loading, using a simple micro-optical technique. The experimental results show that the crazing damage, which is represented by an areal craze density, increases with applied stress and creep time. The craze initiation time-stress envelope for the PMMA at room temperature was determined experimentally. Moreover, the stress and time dependencies of crazing damage were modeled, from which it was possible to predict the creep behavior coupled with growing damage. The predictions are in good agreement with the experimental results.

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References